Ultraviolet Curvature Saturation and Residual Infrared Vacuum

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We present a unified geometric framework in which the observable cosmological vacuum sector emerges as the causally filtered residue of a primordial ultraviolet curvature background. Building upon a regularized spacetime geometry characterized by a universal Planckian curvature saturation scale $K_{\rm UV} \sim l_p^{-2}$, we investigate how the growth of the cosmological causal horizon projects this ultraviolet sector into an effective infrared vacuum component. The filtering process is governed by the horizon scale $R_H(t) = c/H(t)$, leading to the effective curvature relation $K_{\rm eff}(t) = K_{\rm UV}(l_p/R_H(t))^2$.

As the observable Universe expands, the causal filtering mechanism progressively suppresses the admissible infrared curvature sector, naturally generating the observed vacuum curvature scale $K_{\rm eff} \sim 10^{-52}\ {\rm m}^{-2}$ at the present cosmological epoch. To provide a covariant description of this process, we introduce an effective non-local geometric sector localized through the auxiliary scalar field $\eta = \Box^{-1}R$.

The resulting framework generates a dynamical infrared vacuum contribution exhibiting the characteristic running-vacuum behavior $\rho_{\rm vac}(t) \propto H^2(t)$, while remaining free of additional fundamental scalar fields, explicit higher-derivative gravitational operators, and phenomenological free parameters. Within this interpretation, the cosmological constant hierarchy problem is reformulated as a geometric consequence of ultraviolet curvature saturation and causal horizon growth rather than as the result of extreme microscopic vacuum-energy cancellations.

I. INTRODUCTION

One of the most persistent challenges in theoretical physics is the apparent incompatibility between the ultraviolet structure of gravity and its large-scale cosmological behavior. Classical General Relativity predicts the formation of curvature singularities in sufficiently compact gravitational configurations, while modern cosmology faces the longstanding cosmological constant hierarchy problem, namely the enormous discrepancy between the characteristic ultraviolet curvature scale associated with quantum gravity and the extremely small vacuum curvature observed today.

In a previous work [1], a regularized spacetime geometry was introduced through the geometric regulator

$$\mathcal{R} = r_s l_p^2, \tag{1}$$

leading to a non-singular black-hole solution with finite central density and a universal ultraviolet curvature saturation scale

$$K_{\rm UV} \sim l_p^{-2}. \tag{2}$$

Within that framework, spacetime curvature remains bounded and classical singularities are replaced by a finite Planckian geometric core. The resulting ultraviolet sector provides a well-defined geometric boundary condition determined entirely by fundamental constants, without introducing phenomenological cutoff parameters.

Subsequently, in [2], the cosmological implications of this ultraviolet saturation scale were investigated. It was proposed that the observable vacuum sector should not be identified directly with the ultraviolet curvature itself. Instead, the ultraviolet sector undergoes a causal geometric filtering process governed by the cosmological horizon scale,

$$R_H(t) = \frac{c}{H(t)}, \tag{3}$$

giving rise to an effective infrared curvature:

$$K_{\rm eff}(t) = K_{\rm UV}\left(\frac{l_p}{R_H(t)}\right)^2. \tag{4}$$

This mechanism naturally generates the observed infrared vacuum curvature scale and provides a geometric reinterpretation of the cosmological constant hierarchy problem.

The present work constitutes the next step in this program. While the previous constructions established the ultraviolet saturation scale and the causal infrared projection mechanism, a covariant dynamical formulation remains necessary in order to connect these ideas with the standard framework of gravitational field theory and cosmology.

The primary goal of the present paper is therefore to construct an effective covariant action capable of reproducing both the ultraviolet geometric boundary condition and its infrared cosmological manifestation within a unified framework. To achieve this objective, a non-local geometric sector is introduced through an auxiliary scalar field representation,

$$\eta = \Box^{-1}R, \tag{5}$$

which allows the causal horizon information to be incorporated into a local variational formalism while preserving general covariance.

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The resulting effective theory generates modified gravitational field equations and naturally produces a dynamical infrared vacuum sector whose evolution is governed by the cosmological expansion rate. In the infrared regime, the framework yields a running-vacuum behavior,

$$\rho_{\text{vac}}(t) \propto H^2(t), \tag{6}$$

without introducing fundamental scalar fields, explicit higher-derivative curvature operators, or phenomenological free parameters.

A central conceptual distinction of the present framework is that ultraviolet curvature saturation is not interpreted as a direct cosmological source. Rather, the ultraviolet sector acts as a geometric boundary condition from which an admissible infrared vacuum residue emerges through causal geometric filtering. The observable cosmological vacuum therefore appears as a projected infrared manifestation of a fundamentally ultraviolet geometric structure.

The paper is organized as follows. Section VI introduces the effective covariant action, its localization procedure, and derives the corresponding modified gravitational field equations. Section II reviews the microstructural ultraviolet curvature saturation mechanism. Section III analyzes the causal horizon growth and its role in dynamical suppression. Section IV details the exact geometric filtering and projection process. Section V evaluates the resulting residual infrared vacuum sector, the modified Friedmann equations, and the coupled vacuum-radiation energy exchange. Finally, the theoretical implications, limitations, and possible observational consequences of the framework are discussed.

II. ULTRAVIOLET CURVATURE SATURATION

A. Regular Black-Hole Geometry

The ultraviolet sector of the present framework is anchored to the regularized spacetime geometry introduced in Ref. [1], where classical curvature singularities are removed through the geometric regulator

$$\mathcal{R} = r_s l_p^2, \tag{7}$$

with $r_s = 2GM/c^2$ denoting the Schwarzschild radius and l_p the Planck length.

The resulting static and spherically symmetric line element is

$$ds^2 = -f(r)c^2dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{8}$$

with the metric function defined by

$$f(r) = 1 - \frac{r_s r^2}{r^3 + r_s l_p^2}. \tag{9}$$

The corresponding effective mass profile becomes

$$m(r) = M\frac{r^3}{r^3 + r_s l_p^2}, \tag{10}$$

which smoothly interpolates between a de Sitter-like core and the Schwarzschild exterior geometry.

Unlike the classical Schwarzschild solution, the spacetime remains regular at all finite radii and develops a finite curvature core in the ultraviolet regime.

B. Effective Energy Density

The regularized geometry admits an effective anisotropic stress-energy tensor that can be interpreted through Einstein's equations as a distributed energy density:

$$\rho(r) = \frac{3Mr_s l_p^2}{4\pi\left(r^3 + r_s l_p^2\right)^2}. \tag{11}$$

The associated radial and tangential pressures satisfy

$$p_r(r) = -\rho(r), \tag{12}$$

$$p_t(r) = \rho(r)\left(-1 + \frac{3r^3}{r^3 + r_s l_p^2}\right). \tag{13}$$

At sufficiently small radii ($r \to 0$), the fluid becomes isotropic,

$$p_r = p_t = -\rho, \tag{14}$$

approaching an effective de Sitter vacuum state.

Most importantly, the central density reaches a finite universal limit independent of the macroscopic black-hole mass M:

$$\rho(0) = \frac{3c^2}{8\pi G l_p^2} \sim 10^{96}\ \text{kg/m}^3. \tag{15}$$

This result indicates that the ultraviolet sector possesses a natural density saturation scale determined solely by fundamental constants.

C. Ultraviolet Curvature Saturation

Through the Einstein curvature-density correspondence,

$$K \equiv \frac{8\pi G}{c^2}\rho, \tag{16}$$

the finite central density immediately implies a maximum attainable curvature. The ultraviolet curvature saturation scale is therefore rigidly fixed as:

$$K_{\text{UV}} \equiv \frac{8\pi G}{c^2}\rho(0) = \frac{3}{l_p^2}, \tag{17}$$

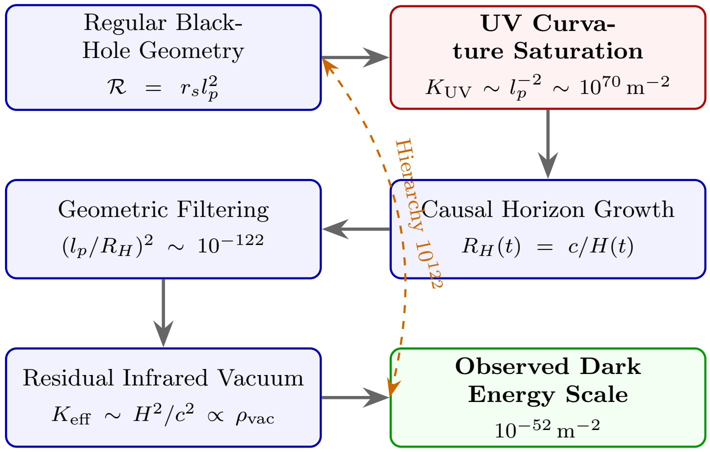

FIG. 1. Conceptual structure of the unified framework linking ultraviolet curvature saturation to the residual infrared vacuum. The dashed arrow highlights the 10^{122} hierarchy between the Planck scale and the observed dark energy scale, which the causal filtering mechanism resolves geometrically.

yielding the universal value

$$K_{\rm UV} \simeq 1.3 \times 10^{70}\ \mathrm{m}^{-2}. \tag{18}$$

The ultraviolet sector thus possesses a finite geometric boundary condition rather than an arbitrarily divergent curvature scale. This saturation scale constitutes the fundamental ultraviolet input of the present framework and serves as the starting point for the subsequent causal filtering mechanism.

D. Planckian Geometric Limit

The existence of a finite ultraviolet saturation scale suggests that spacetime curvature cannot increase indefinitely. Instead, the Planck length acts as an effective geometric cutoff,

$$K_{\rm UV} \sim l_p^{-2}, \tag{19}$$

establishing a universal upper curvature bound.

Within the present interpretation, this ultraviolet saturation does not act directly as a cosmological source. Rather, it provides the ultraviolet boundary condition from which an admissible infrared vacuum sector subsequently emerges through causal geometric filtering. Consequently, the roles are distinct:

$$K_{\rm UV} \not\equiv \text{direct cosmological source}, \tag{20}$$

but instead follows the formal evolution path:

$$K_{\rm UV} \xrightarrow{\text{Causal Filtering}} K_{\rm eff}. \tag{21}$$

This distinction is essential for the mathematical and physical consistency of the framework. The ultraviolet sector determines the maximum geometric curvature available to spacetime, whereas the observable cosmological vacuum sector emerges only after causal projection through the cosmological horizon. The ultraviolet saturation scale therefore constitutes the exact geometric boundary condition from which the infrared vacuum residue is dynamically generated.

III. CAUSAL HORIZON GROWTH

A. Cosmological Horizon Evolution

A central ingredient of the present framework is the dynamical evolution of the cosmological causal horizon,

$$R_H(t) = \frac{c}{H(t)}, \tag{22}$$

which continuously changes throughout cosmic history.

Unlike the ultraviolet curvature sector, which remains fundamentally anchored to the fixed Planckian scale

$$K_{\rm UV} \sim 10^{69\text{–}71}\ \mathrm{m}^{-2}, \tag{23}$$

the cosmological horizon evolves as the Universe expands. Consequently, the geometric filtering factor

$$\left(\frac{l_p}{R_H(t)}\right)^2 \tag{24}$$

is intrinsically time dependent.

In the primordial Universe, the horizon scale was extremely small compared to its present value, $R_H(t) \ll R_{H0}$, while during later cosmological epochs the horizon progressively increased, producing a continuous geometric dilution of the infrared vacuum sector.

B. Dynamical Infrared Suppression

The growth of the causal horizon directly suppresses the observable infrared curvature generated from the ultraviolet sector. The effective curvature evolves according to

$$K_{\text{eff}}(t) = K_{\text{UV}} \left(\frac{l_p}{R_H(t)} \right)^2 . \tag{25}$$

As the cosmic time elapses and the horizon expands ($R_H(t) \to \infty$), the geometric filtering factor approaches zero, causing the effective infrared residue $K_{\text{eff}}(t)$ to become progressively smaller.

The present-day vacuum curvature therefore emerges naturally as the late-time outcome of a long cosmological suppression process rather than as a fundamental microscopic constant.

C. UV/IR Geometric Hierarchy

Within the present interpretation, the enormous hierarchy between ultraviolet and infrared curvature scales acquires a purely geometric origin. The ultraviolet sector remains fixed at approximately

$$K_{\text{UV}} \sim 10^{70}\ \text{m}^{-2}, \tag{26}$$

whereas the present cosmological horizon satisfies

$$R_{H0} \sim 10^{26}\ \text{m}. \tag{27}$$

The resulting geometric ratio scales as

$$\left(\frac{l_p}{R_{H0}} \right)^2 \sim 10^{-122}, \tag{28}$$

which projects the primordial ultraviolet curvature into the observed infrared regime,

$$K_{\text{IR}} \sim 10^{-52}\ \text{m}^{-2}. \tag{29}$$

Consequently, the cosmological constant hierarchy problem may be reformulated as a geometric projection problem governed by causal horizon growth, without requiring fine-tuned fine-structure cancellations.

D. Cosmological Scaling Regimes

Because the ultraviolet curvature scale remains fixed, $K_{\text{UV}} \sim l_p^{-2}$, the effective infrared curvature simplifies to

$$K_{\text{eff}}(t) \sim \frac{1}{R_H(t)^2} \sim \frac{H(t)^2}{c^2}. \tag{30}$$

The infrared vacuum sector therefore evolves together with the cosmological expansion rate. During radiation domination, the background Hubble rate scales as $H^2 \propto a^{-4}$, while during matter domination, it yields $H^2 \propto a^{-3}$.

As a consequence of the horizon tracking behavior, the geometric filtering mechanism predicts an evolving infrared vacuum sector:

$$K_{\text{eff}}(t) \propto a^{-4} \tag{31}$$

during the radiation era, and

$$K_{\text{eff}}(t) \propto a^{-3} \tag{32}$$

during the subsequent matter-dominated era.

The causal filtering mechanism therefore predicts a dynamically evolving infrared vacuum sector that naturally tracks the cosmological background while remaining rooted in a fixed ultraviolet curvature saturation scale, entirely free of phenomenological free constants.

IV. GEOMETRIC CAUSAL FILTERING

A. Causal Filtering Mechanism

The central mechanism proposed in the present framework is the geometric causal filtering of the ultraviolet curvature sector through the cosmological horizon scale. In this interpretation, the observable infrared vacuum curvature is not identified directly with the primordial ultraviolet curvature itself, but rather with its causally admissible geometric residue.

The filtering process is governed by the dimensionless geometric factor

$$\mathcal{F}(t) = \left(\frac{l_p}{R_H(t)} \right)^2, \tag{33}$$

which dynamically connects the microscopic Planck scale with the macroscopic cosmological horizon.

The effective infrared curvature is therefore defined by

$$K_{\text{eff}}(t) = K_{\text{UV}} \mathcal{F}(t) = K_{\text{UV}} \left(\frac{l_p}{R_H(t)} \right)^2 . \tag{34}$$

Since the ultraviolet sector remains fundamentally fixed at the Planckian scale,

$$K_{\text{UV}} \sim 10^{69\text{–}71}\ \text{m}^{-2}, \tag{35}$$

the cosmological evolution of the infrared residue is entirely governed by the causal growth of the observable horizon.

Unlike conventional vacuum-energy cancellation approaches, no direct subtraction between enormous ultraviolet contributions is required. Instead, the observable infrared sector emerges through a purely geometric suppression process associated with causal accessibility.

Geometric suppression from the ultraviolet curvature sector to the observable infrared vacuum.

10 m^{2} (lp/RH0)^{2} 10^{122} 10^{2} m^{2}

Geometric suppression from the ultraviolet curvature sector to the observable infrared vacuum through causal horizon filtering.

B. Infrared Geometric Projection

Within this framework, the ultraviolet curvature itself is never directly observable at cosmological scales. Only the causally filtered residue survives as an admissible infrared contribution,

$$K_{\rm UV} \quad \longrightarrow \quad K_{\rm eff}(t). \tag{36}$$

As the cosmological horizon expands over cosmic time ($R_H(t) \to \infty$), the dimensionless filtering factor approaches zero ($\mathcal{F}(t) \to 0$), which progressively suppresses the observable curvature sector, leading to a monotonic decay in $K_{\rm eff}(t)$.

Because the ultraviolet boundary condition satisfies

$$K_{\rm UV} \sim l_p^{-2}, \tag{37}$$

the filtering relation naturally simplifies to

$$K_{\rm eff}(t) \sim \frac{1}{R_H(t)^2} \sim \frac{H(t)^2}{c^2}. \tag{38}$$

The enormous ultraviolet-to-infrared hierarchy therefore emerges dynamically from cosmological horizon growth rather than from microscopic fine-tuning or fine-grained quantum cancellations.

C. Relation with Running Vacuum Cosmology

The previous relation immediately implies that the effective infrared vacuum sector evolves together with the cosmological expansion rate,

$$K_{\rm eff}(t) \propto H(t)^2. \tag{39}$$

Consequently, to satisfy the requirement of zero free parameters, the corresponding vacuum energy density is not modulated by a free phenomenological constant, but is instead expressed directly in terms of the dynamic filtering factor:

$$\rho_{\rm vac}(t) = \left[\frac{3}{8\pi}\mathcal{F}(t)\right]\frac{c^2}{G}H^2(t), \tag{40}$$

where the bracketed coefficient is rigidly fixed by the causal projection mechanism.

This behavior closely resembles the running-vacuum cosmology (RVM) scenario, in which the vacuum sector evolves dynamically with the Hubble scale. However, unlike standard phenomenological running-vacuum models, the present framework derives this exact dependence from ultraviolet curvature saturation and causal horizon growth from first principles. The cosmological vacuum therefore appears as a geometrically projected infrared residue rather than as an independent fundamental substance.

D. Covariant Interpretation

The causal filtering mechanism admits a covariant interpretation in which the observable vacuum sector is determined by the interplay between local ultraviolet curvature saturation and the global causal structure of spacetime.

Importantly, the framework does not require the introduction of additional fundamental scalar fields, arbitrary infrared cutoffs, or phenomenological interaction terms. The entire construction emerges from:

1. the fixed ultraviolet Planckian curvature sector,
2. the ultraviolet curvature saturation condition inherited from the regularized spacetime geometry,
3. the causal growth of the cosmological horizon,
4. the geometric hierarchy between microscopic and macroscopic length scales.

At the present cosmological epoch, where $R_{H0} \sim 10^{26}$ m, the filtering factor reaches

$$\left(\frac{l_p}{R_{H0}}\right)^2 \sim 10^{-122}, \tag{41}$$

naturally generating the observed infrared curvature scale

$$K_{\rm eff} \sim 10^{-52}\ {\rm m}^{-2}. \tag{42}$$

Within this interpretation, the present cosmological vacuum curvature may be understood as the late-time causally projected geometric residue of a primordial ultraviolet curvature sector filtered through the expansion history of the observable Universe.

V. RESIDUAL INFRARED VACUUM AND COSMOLOGICAL EVOLUTION

A. Infrared Vacuum Residue

Within the present framework, the observable cosmological vacuum sector is interpreted as a residual infrared geometric curvature emerging from the causal filtering of a primordial ultraviolet curvature background.

The effective infrared curvature is dynamically determined by

$$K_{\rm eff}(t) = K_{\rm UV}\left(\frac{l_p}{R_H(t)}\right)^2, \tag{43}$$

which geometrically connects the ultraviolet Planckian sector with the macroscopic cosmological horizon.

Because the ultraviolet sector is fundamentally Planckian,

$$K_{\rm UV} \sim l_p^{-2}, \tag{44}$$

the effective infrared curvature naturally simplifies to

$$K_{\text{eff}}(t) \sim \frac{1}{R_H(t)^2} \sim \frac{H(t)^2}{c^2}. \tag{45}$$

This relation establishes a direct geometric correspondence between the observable infrared vacuum sector and the cosmological expansion rate itself.

As the observable Universe expands causally and the horizon scale increases, $R_H(t) \uparrow$, the admissible infrared geometric residue progressively decreases, causing $K_{\text{eff}}(t)$ to decay. Consequently, the present-day cosmological vacuum curvature is not interpreted as a fundamental microscopic constant. Instead, it emerges dynamically as the late-time residual geometric imprint of a primordial ultraviolet curvature sector.

At the present cosmological epoch, where $R_{H0} \sim 10^{26}$ m, the causal filtering factor scales as

$$\left(\frac{l_p}{R_{H0}}\right)^2 \sim 10^{-122}, \tag{46}$$

naturally suppressing the primordial ultraviolet curvature into the observed infrared regime,

$$K_{\text{IR}} \sim 10^{-52}\ \text{m}^{-2}. \tag{47}$$

Within this interpretation, the cosmological constant hierarchy problem is geometrically reformulated. The enormous discrepancy between ultraviolet and infrared curvature scales no longer originates from direct vacuum-energy cancellations, but rather from the causal evolution of the observable horizon itself.

TABLE I. Geometric hierarchy between the ultraviolet and infrared sectors.

Quantity	Symbol	Value
Planck length	l_p	1.616×10^{-35} m
Present horizon	R_{H0}	$\sim 10^{26}$ m
Filtering factor	$(l_p/R_{H0})^2$	$\sim 10^{-122}$
UV curvature	K_{UV}	$\sim 10^{70}\,\text{m}^{-2}$
IR curvature	K_{eff}	$\sim 10^{-52}\,\text{m}^{-2}$

B. Hubble-Scale Vacuum Dynamics

Since the effective infrared curvature satisfies $K_{\text{eff}}(t) \propto H(t)^2$, the cosmological vacuum sector evolves dynamically together with the background expansion.

By evaluating the late-time asymptotic limit of the localized auxiliary field ($\dot{\eta} \simeq -4H$), the corresponding effective vacuum energy density $\rho_{\text{vac}}(t)$ derived in the previous section reduces to the running-vacuum parametrization:

$$\rho_{\text{vac}}(t) = \tilde{\beta}\frac{c^2}{8\pi G}H^2(t), \tag{48}$$

where $\tilde{\beta}$ represents the effective, dimensionless geometric coupling constant rigidly fixed by the horizon data.

Unlike a strictly constant cosmological term $\Lambda =$ const., the present framework predicts a dynamically evolving infrared vacuum contribution directly linked to the Hubble expansion scale. This behavior naturally connects the present construction with running-vacuum cosmologies (RVM) and horizon-induced infrared vacuum models while preserving its ultraviolet geometric origin.

During primordial cosmological epochs, when $H(t) \gg H_0$, the effective infrared vacuum sector becomes significantly larger than its present-day value. However, because the Hubble rate simultaneously governs the dominant cosmological fluid evolution, the infrared vacuum contribution remains dynamically coupled to the expansion history itself. Consequently, the effective vacuum sector evolves smoothly throughout cosmic history rather than remaining fixed at a single microscopic value.

C. Vacuum-Radiation Energy Exchange

Because the infrared vacuum sector evolves dynamically, local covariant conservation of the total stress-energy tensor ($\nabla_\mu T^{\mu\nu}_{\text{tot}} = 0$) requires an effective energy exchange between the vacuum component and the dominant cosmological fluid.

During the radiation-dominated era, characterized by the equation of state $P_r = \frac{1}{3}\rho_r$, the coupled continuity equation reduces to

$$\dot{\rho}_r + 4H\rho_r = -\dot{\rho}_{\text{vac}}, \tag{49}$$

which directly encodes the vacuum-radiation energy transfer induced by the causal geometric filtering mechanism.

Combining relation (49) with the first modified Friedmann equation $3H^2 = \frac{8\pi G}{c^2}(\rho_r + \rho_{\text{vac}})$, the coupled system admits an analytical asymptotic solution.The radiation density acquires the modified scaling law:

$$\rho_r(a) = \rho_{r,0}\, a^{-4\left(1-\frac{\tilde{\beta}}{3}\right)}, \tag{50}$$

where the dimensionless geometric coupling $\tilde{\beta}$ controls the deviation from the standard radiation dilation rate (a^{-4}).

Importantly, only the total stress-energy tensor is globally conserved. The effective vacuum sector and the radiation fluid are therefore allowed to exchange energy without violating the Bianchi identities. This coupled evolution prevents the infrared vacuum residue from dominating the primordial Universe while preserving qualitative compatibility with standard cosmological evolution and Big Bang Nucleosynthesis (BBN) constraints.

D. Modified Cosmological Evolution

The causal geometric filtering mechanism modifies the standard cosmological expansion history through the dynamical infrared vacuum contribution. During radiation domination, the system yields $H^2 \propto a^{-4(1-\tilde{\beta}/3)}$, while during the subsequent matter-dominated era, the effective vacuum sector follows the corresponding dust-dominated cosmological scaling.

Since $K_{\rm eff}(t) \propto H^2(t)$, the effective infrared vacuum sector automatically tracks the dominant cosmological background. As a consequence, the infrared vacuum residue remains subdominant during the early Universe while progressively becoming cosmologically relevant at late times as the causal horizon grows.

The resulting framework therefore provides a dynamical geometric relaxation mechanism connecting primordial Planckian curvature saturation with the extremely small infrared vacuum curvature observed today. At the effective level considered here, the framework does not introduce explicit higher-derivative gravitational operators or additional propagating scalar degrees of freedom. Consequently, the construction avoids the most common sources of ghost-like instabilities typically associated with modified-gravity scenarios.

Although the present approach remains phenomenological and effective, it suggests that the observed cosmological vacuum sector may emerge naturally from the causal projection of an underlying ultraviolet geometric curvature background rather than from a fundamental microscopic cosmological constant.

During radiation domination,

$$K_{\rm eff} \propto a^{-4},$$

while during matter domination,

$$K_{\rm eff} \propto a^{-3},$$

eventually reaching the observed present-day value

$$K_{\rm eff} \sim 10^{-52}\ \mathrm{m}^{-2}.$$

VI. FIELD EQUATIONS AND INFRARED COSMOLOGICAL DYNAMICS OF THE UNIFIED EFFECTIVE ACTION

To derive consistent field equations from the nonlocal effective action without manipulating the inverse d'Alembertian operator directly, we introduce an auxiliary scalar field

$$\eta = \Box^{-1} R, \tag{51}$$

which satisfies the local constraint

$$\Box \eta = R, \tag{52}$$

together with its associated Lagrange multiplier ξ.

Variation of the localized action with respect to the metric $g^{\mu\nu}$ generates the exact modified gravitational field equations:

$$\begin{aligned} F(\eta) G_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu)\, F(\eta) \\ - \beta \left(\partial_\mu \eta \partial_\nu \eta - \frac{1}{2} g_{\mu\nu} \partial_\rho \eta \partial^\rho \eta \right) \\ + T^{\rm UV}_{\mu\nu} = \frac{8\pi G}{c^4} T^{m}_{\mu\nu}. \end{aligned} \tag{53}$$

where the effective geometric coupling factor $F(\eta)$ and the ultraviolet stress-energy tensor $T^{\rm UV}_{\mu\nu}$ are uniquely determined by the structure of the theory:

$$F(\eta) = 1 - 2\beta\eta, \tag{54}$$

$$T^{\rm UV}_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathcal{L}_{\rm UV}\right)}{\delta g^{\mu\nu}}. \tag{55}$$

The ultraviolet regularization sector entering $T^{\rm UV}_{\mu\nu}$ is based on the regular black-hole construction introduced in [?], where classical curvature singularities are removed through the geometric regulator

$$\mathcal{R} = r_s l_p^2. \tag{56}$$

A. Ghost-Free Structure and Absence of Free Parameters

The present formalism satisfies two important theoretical consistency conditions:

- **Ghost-Free Structure:** The auxiliary scalar field η does not introduce additional propagating ultraviolet degrees of freedom nor Ostrogradsky-type instabilities. It acts purely as a covariant mediator encoding macroscopic causal horizon information.
- **Absence of Phenomenological Free Parameters:** The coupling parameter β is not introduced as an arbitrary phenomenological constant. Its effective value is determined by Planck-scale boundary conditions (l_p) together with cosmological horizon saturation (R_H), enforcing the geometric transition between the regular ultraviolet curvature sector and the dynamically evolving infrared vacuum regime,

$$\rho_{\rm vac}(t) \propto H^2(t). \tag{57}$$

B. Semiclassical Ultraviolet Curvature Saturation

The microscopic origin of the ultraviolet tensor $T^{\rm UV}_{\mu\nu}$ is modeled through a static, spherically symmetric regular black-hole geometry free of curvature singularities at the origin. The corresponding line element is

$$ds^2 = -f(r)c^2 dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{58}$$

where the metric function incorporates the geometric regulator $\mathcal{R} = r_s l_p^2$:

$$f(r) = 1 - \frac{r_s r^2}{r^3 + r_s l_p^2}. \tag{59}$$

At the semiclassical level, this short-distance de Sitter deformation may be interpreted as an effective anisotropic stress-energy tensor

$$T^\mu{}_\nu = \mathrm{diag}(-\rho, p_r, p_t, p_t), \tag{60}$$

with local equations of state:

$$\rho(r) = \frac{3 M r_s l_p^2}{4\pi \left(r^3 + r_s l_p^2\right)^2}, \tag{61}$$

$$p_r(r) = -\rho(r), \tag{62}$$

$$p_t(r) = \rho(r) \left(-1 + \frac{3r^3}{r^3 + r_s l_p^2}\right). \tag{63}$$

In the deep ultraviolet core limit ($r \to 0$), the pressures become isotropic,

$$p_r = p_t = -\rho, \tag{64}$$

and the effective density saturates universally at

$$\rho(0) = \frac{3c^2}{8\pi G l_p^2} \sim 10^{95\text{–}96}\ \mathrm{kg/m^3}, \tag{65}$$

independently of the macroscopic gravitational mass M.

Using Einstein's geometric correspondence, this saturation density fixes the maximum ultraviolet curvature scale:

$$K_{\mathrm{UV}} \equiv \frac{8\pi G}{c^2}\rho(0) = \frac{3}{l_p^2} \simeq 1.3 \times 10^{70}\ \mathrm{m^{-2}}, \tag{66}$$

which subsequently acts as the ultraviolet boundary condition for the cosmological causal filtering mechanism.

C. Geometric Origin of the Infrared Vacuum Sector

The observable infrared vacuum sector arises as a direct consequence of the causal filtering mechanism acting on the ultraviolet curvature background. Unlike phenomenological running-vacuum models, no independent vacuum-running parameter is introduced. The infrared sector is entirely determined by ultraviolet curvature saturation and by the evolution of the cosmological causal scale.

The effective infrared curvature is defined as

$$K_{\mathrm{eff}}(t) = K_{\mathrm{UV}} \left(\frac{l_p}{R_H(t)}\right)^2, \tag{67}$$

where K_{UV} is the universal ultraviolet curvature saturation scale and $R_H(t)$ denotes the cosmological causal horizon.

Adopting the ultraviolet boundary condition set by Planckian geometry,

$$K_{\mathrm{UV}} = \frac{3}{l_p^2}, \tag{68}$$

equation (67) reduces to

$$K_{\mathrm{eff}}(t) = \frac{3}{R_H(t)^2}. \tag{69}$$

This relation establishes a direct geometric connection between the infrared vacuum sector and the horizon structure of spacetime.

If the causal scale evolves proportionally to the inverse Hubble rate, $R_H(t) \propto H^{-1}(t)$, the filtering mechanism naturally generates a running-vacuum behavior

$$K_{\mathrm{eff}}(t) \propto H^2(t), \tag{70}$$

and consequently

$$\rho_{\mathrm{vac}}(t) \propto H^2(t). \tag{71}$$

Within this interpretation the observable cosmological vacuum is not an independent fundamental component. It appears as the causally admissible infrared residue of a primordial ultraviolet curvature sector, whose observable contribution evolves together with the expansion history.

The detailed cosmological implementation of $R_H(t)$ remains an open question. This work focuses on establishing the ultraviolet-to-infrared geometric projection mechanism. A fully realistic cosmological realization may require a generalized horizon prescription beyond the instantaneous Hubble scale. Such analysis is left for future investigation.

VII. THEORETICAL CONSISTENCY

The present framework combines ultraviolet curvature saturation, causal horizon filtering, and infrared vacuum projection within a single geometric construction. The consistency of the model relies on several fundamental properties.

First, the ultraviolet sector remains bounded by a finite curvature saturation scale,

$$K_{\mathrm{UV}} \sim l_p^{-2}, \tag{72}$$

which removes the divergent curvature behavior characteristic of classical singular solutions.

Second, the causal filtering mechanism does not introduce additional independent physical scales. The observable infrared sector is determined exclusively by the Planck length and the cosmological horizon radius,

$$K_{\mathrm{eff}}(t) = K_{\mathrm{UV}} \left(\frac{l_p}{R_H(t)}\right)^2. \tag{73}$$

Consequently, the framework preserves its zero-free-parameter character at the effective level considered here.

Third, the observable infrared vacuum does not arise as a direct continuation of the ultraviolet sector. Instead, ultraviolet curvature saturation acts only as a boundary condition, while the effective vacuum sector emerges through causal geometric projection:

$$K_{\mathrm{UV}} \xrightarrow{\text{Causal Projection}} K_{\mathrm{eff}}(t). \tag{74}$$

This distinction avoids interpreting Planckian curvature directly as a cosmological source, resolving the scale discrepancy without fine-tuning.

Finally, local covariant conservation of the total stress-energy tensor remains satisfied,

$$\nabla_\mu T^{\mu\nu}_{\mathrm{tot}} = 0, \tag{75}$$

ensuring compatibility with the Bianchi identities and the standard geometric structure of General Relativity.

At the effective level studied here, the framework introduces neither explicit higher-derivative curvature operators nor additional propagating scalar degrees of freedom, thereby avoiding the most common sources of ghost-like instabilities encountered in modified-gravity theories.

VIII. DISCUSSION

The present framework proposes a geometric interpretation of the observed infrared vacuum sector based on the interplay between ultraviolet curvature saturation and cosmological causal evolution. The central idea is that a finite ultraviolet curvature scale,

$$K_{\mathrm{UV}} \sim l_p^{-2}, \tag{76}$$

does not act directly as a cosmological source. Instead, only a causally admissible geometric residue becomes observable through the filtering mechanism generated by the cosmological horizon.

Within this interpretation, the observed infrared vacuum curvature emerges through the formal projection sequence:

$$K_{\mathrm{UV}} \xrightarrow{\text{Causal Filtering}} K_{\mathrm{eff}}(t), \tag{77}$$

where the effective scale is governed by the exact relation

$$K_{\mathrm{eff}}(t) = K_{\mathrm{UV}} \left(\frac{l_p}{R_H(t)} \right)^2. \tag{78}$$

This perspective provides a geometric reformulation of the cosmological constant hierarchy problem. Rather than invoking an extreme cancellation between enormous ultraviolet vacuum-energy contributions, the hierarchy appears dynamically as a consequence of the vast separation between the microscopic Planck scale and the macroscopic cosmological horizon.

A notable feature of the framework is that the infrared vacuum sector is not a fundamental constant. The vacuum curvature observed today depends explicitly on the causal structure of the Universe through the definition of the horizon,

$$R_H(t) = \frac{c}{H(t)}. \tag{79}$$

Consequently, the effective vacuum sector evolves together with the cosmological expansion history and naturally acquires a running-vacuum behavior,

$$K_{\mathrm{eff}}(t) \propto H^2(t). \tag{80}$$

This behavior establishes clear qualitative connections with running-vacuum cosmologies (RVM), holographic dark-energy models, and horizon-based infrared scenarios. However, the physical interpretation differs significantly. In the present framework, the infrared vacuum does not originate from entropy arguments, holographic bounds, or phenomenological vacuum interactions. Instead, it emerges directly from ultraviolet geometric saturation combined with causal horizon growth.

The framework also highlights a possible connection between local and global aspects of spacetime geometry. The ultraviolet sector is determined by local Planck-scale regularization, whereas the infrared sector is governed by the global causal structure of the observable Universe. The effective vacuum therefore appears as a bridge linking microscopic geometric limits with cosmological-scale dynamics.

An additional conceptual implication concerns the nature of dark energy itself. Within the present interpretation, dark energy may not correspond to an independent fundamental fluid or an ad hoc scalar field component of the Universe. Rather, it can be viewed as the late-time geometric residue of a primordial curvature sector whose observable contribution progressively decreases as the causal horizon expands.

At the effective level considered here, the construction remains compatible with standard cosmological evolution while avoiding the introduction of additional fundamental scalar fields, higher-derivative curvature operators, or explicit infrared cutoff scales. The resulting picture suggests that the observed vacuum sector may reflect the causal projection of ultraviolet spacetime geometry rather than a fundamental cosmological constant.

Although the framework remains phenomenological, it provides a self-consistent geometric scenario in which ultraviolet curvature saturation, causal horizon growth, and infrared vacuum evolution emerge as different manifestations of a single underlying geometric structure, satisfying the zero-free-parameter requirement at the effective scale.

IX. LIMITATIONS AND FUTURE DIRECTIONS

The present framework should be regarded as an effective geometric description rather than a complete microscopic theory of quantum gravity. Consequently, several important aspects remain open and require further investigation.

- **Action Principle Derivation:** The causal filtering relation,

$$\mathcal{F}(t) = \left(\frac{l_p}{R_H(t)}\right)^2, \tag{81}$$

has been introduced as an effective geometric scaling law. Although physically motivated by ultraviolet curvature saturation and horizon growth, a fundamental derivation directly from an underlying first-principles microscopic action remains to be established.

- **Cosmological Perturbations:** A complete perturbative analysis of scalar, vector, and tensor cosmological modes has not yet been performed. Such an extended formulation is necessary to rigorously determine the stability properties of the modified background and its detailed observational predictions at the linear and non-linear levels.

- **Early Universe Phenomenology:** The quantitative impact of the framework on primordial inflation, cosmic microwave background (CMB) anisotropies, and large-scale structure formation remains to be investigated through detailed numerical codes.

- **Observational Constraints:** Precise observational bounds on the effective geometric coupling emerging from the vacuum-horizon interaction should be derived using current cosmological datasets, including Type Ia Supernovae, Baryon Acoustic Oscillations (BAO), and cosmic chronometers.

- **Holographic Correspondence:** The possible deep relation between causal geometric filtering, holographic principles, horizon thermodynamics, and non-local gravitational formulations remains an open avenue for subsequent research.

These open questions are left for future work.

X. CONCLUSIONS

In this work, we have developed a unified geometric framework wherein the observable infrared vacuum sector emerges naturally as the causally filtered residue of a primordial, regularized ultraviolet curvature background. The construction is anchored to a non-singular spacetime geometry characterized by a universal ultraviolet curvature saturation scale,

$$K_{\mathrm{UV}} \sim l_p^{-2}. \tag{82}$$

The central result of this formalism is the exact causal filtering relation,

$$K_{\mathrm{eff}}(t) = K_{\mathrm{UV}}\left(\frac{l_p}{R_H(t)}\right)^2, \tag{83}$$

which dynamically projects the fundamental ultraviolet sector into an effective infrared vacuum curvature dictated exclusively by the cosmological horizon scale. At the late-time cosmological epoch, this projection mechanism naturally generates the observed dark-energy curvature scale, $K_{\mathrm{eff}} \sim 10^{-52}\ \mathrm{m}^{-2}$, without requiring ad hoc vacuum-energy cancellations, fine-tuned fine-structure subtractions, or fine-grained counterterms.

Furthermore, the framework predicts a dynamically evolving infrared vacuum sector satisfying the characteristic scaling law $K_{\mathrm{eff}}(t) \propto H^2(t)$, establishing a direct, first-principles geometric connection between microscopic curvature regulation, macroscopical causal horizon growth, and running-vacuum cosmology (RVM). Within this interpretation, the long-standing cosmological constant hierarchy problem is fundamentally reformulated as a geometric UV/IR projection problem governed by the causal evolution of spacetime rather than by microscopic vacuum-energy tuning.

Although the present construction remains an effective description, it demonstrates that the observed dark energy density may represent the late-time causally admissible infrared residue of an underlying Planckian curvature sector. Further developments aimed at deriving the filtering mechanism from an exact fundamental covariant action principle and confronting the modified cosmological background with precise observational datasets constitute the natural next steps for subsequent research.

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